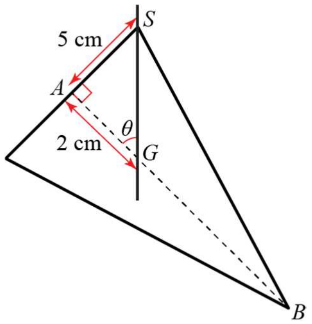
Exercise 6A

1



The diagram shows the equilibrium position with the centre of mass G, vertically below the point of suspension S.

As $AG = \frac{1}{4}h$ for a cone

$\therefore AG = 2\text{cm}$

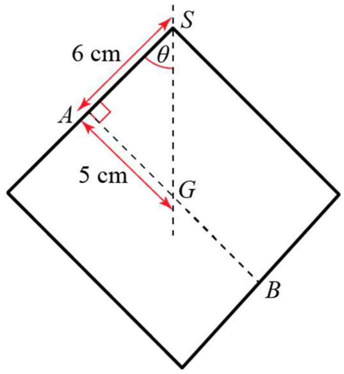
Also the radius $AS = 5\text{cm}$.

Let the angle between the vertical and the axis be θ.

Then from $\Delta ASG, \tan\theta = \frac{5}{2}$

$\therefore \theta = 68^\circ$ (to the nearest degree)

2



The diagram shows the equilibrium position with the centre of mass G below the point of suspension S.

As $AG = \frac{1}{2}h$ for a uniform cylinder

$\therefore AG = 5\text{cm}$

Also the radius $AS = 6\text{cm}$.

The angle between the vertical and the circular base of the cylinder is θ.

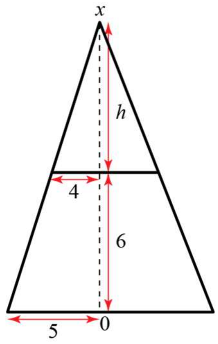
From $\Delta ASG, \tan\theta = \frac{5}{6}$

$\therefore \theta = 40^\circ$ **(to the nearest degree)**

3 The distance from the centre of mass to the base is $\frac{1}{2}r$ from the centre. The angle between the axis of the shell and the downward vertical when the shell is in equilibrium

$\tan\theta = \frac{r}{\frac{1}{2}r} = 2 \Rightarrow \theta = \arctan 2 \approx 63.4^\circ$ (3 s.f.).

4 a



From similar triangles

$$\frac{h}{h+6}=\frac{4}{5}$$

$$\therefore 5h=4h+24$$

i.e. $h=24$

Centre of mass lies at the axis of symmetry OX.

Shape	Mass	Mass ratios	Position of centre of mass i.e. distance from O
Large cone	$\frac{1}{3}\pi\rho\times 5^2\times 30$	125	$\frac{30}{4}=7.5$
Small cone	$\frac{1}{3}\pi\rho\times 4^2\times 24$	64	$6+\frac{24}{4}=12$
Frustum	$\frac{250\pi}{3}\rho-128\pi\rho$	61	$\bar{x}$

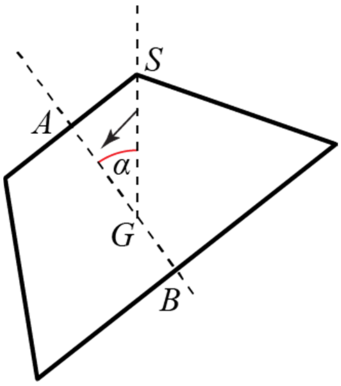
Take moments about O

$$125\times 7.5-64\times 12=61\bar{x}$$

$$\therefore 169.5=61\bar{x}$$

$$\therefore \bar{x}=2.78\ (3\text{ s.f.})\left(\text{or } \frac{339}{122}\right)$$

b



In equilibrium the centre of mass G lies vertically below the point of suspension S.

Let the required angle be α.

AS is smaller radius = 4 cm

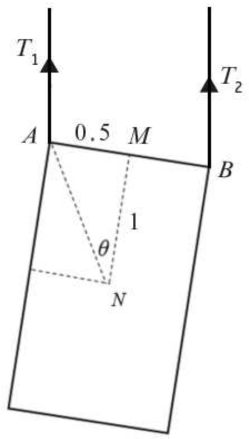
$AG=6-2.78=3.22\text{ cm}\ (3\text{ s.f.})$

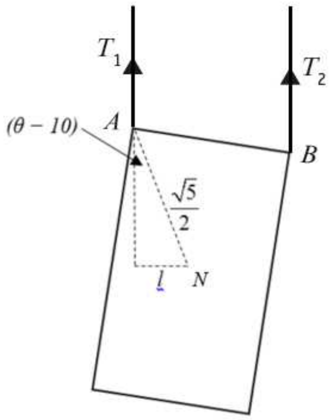
$$\tan\alpha=\frac{AS}{AG}=\frac{4}{3.22}$$

$$\therefore \alpha=51°\ (\text{to the nearest degree})$$

5 $\tan\theta = \frac{\frac{1}{2}}{1} \Rightarrow \theta = \tan^{-1}\left(\frac{1}{2}\right) = 26.565...$

$AN = \sqrt{1^2 + \left(\frac{1}{2}\right)^2} = \frac{\sqrt{5}}{2}$





$\sin(\theta - 10) = \frac{l}{\sqrt{\frac{5}{2}}} \Rightarrow l = 0.3187...$

perpendicular distance, x, of B from A is

$\cos 10 = \frac{x}{1} \Rightarrow x = 0.9848...$

Taking moments about A

$0.3187 \times 2g = 0.9848T_2$

$T_2 = 6.34$ N (3 s.f.)

Since $T_1 + T_2 = 2g$

$T_1 = 13.3$ N (3 s.f.)

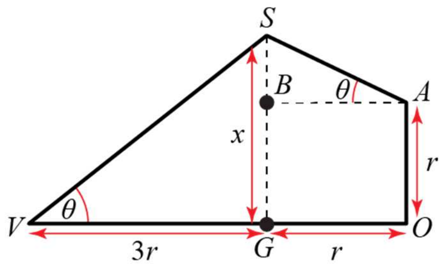
6 a The volume of the uniform solid is
$V = \pi \times 5^2 \times 10 - \frac{2}{3} \times \pi \times 3^3 = 232\pi \text{ cm}^3$. The centre of mass of the solid can be found by taking moments about point O $\pi \times 5^2 \times 10 \times 5 - \frac{2}{3} \times \pi 3^3 \times \frac{3}{8} \times 3 = 232\pi\bar{x}$
$\Rightarrow \bar{x} \approx 5.30$ cm horizontally from the point O. Note that we will want to use metric units from this point. Resolving forces vertically gives $T_1 + T_2 = Mg$. Taking the moments about the point A gives

$$T_2 \times 0.1 = Mg\bar{x} \Rightarrow T_1 = Mg(1 - 10\bar{x})$$
$$= 232 \times 10^{-6} \times 10 \times (1 - 10 \times 0.053)$$
$$\approx 1.07 \times 10^{-3} \text{ N (3 s.f.) where we have taken } g = 9.8.$$

Using $T_1 + T_2 = Mg$ we then obtain that
$T_2 \approx 1.21 \times 10^{-3}$ N (3 s.f.).

b As the horizontal from the point A will be going through the centre of mass and the radius of the cylinder is 5 cm, the angle is $\tan\theta = \frac{\bar{x}}{5} \approx 1.06 \Rightarrow \theta \approx 46.7^\circ$.

7 a



In equilibrium the centre of mass G lies below the point of suspension S. Let distance $SG = x$.
O is the centre of the base of the cone and V is its vertex.
A and B are shown on the diagram.

$$\tan\theta = \frac{x}{3r} \text{ (from } \Delta VSG)$$

$$\text{Also } \tan\theta = \frac{x - r}{r} \text{ (from } \Delta ABS)$$

$$\therefore \frac{x}{3r} = \frac{x - r}{r}$$
$$\therefore x = 3x - 3r$$
$$\therefore 2x = 3r$$
$$\therefore x = \frac{3r}{2}$$
$$\therefore \tan\theta = \frac{1}{2}$$

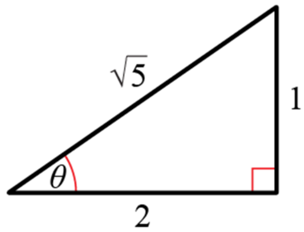
7 b Resolve vertically for the forces acting on the cone:

$2T\sin\theta = mg$

$\therefore T = \dfrac{mg}{2\sin\theta}$

As $\tan\theta = \dfrac{1}{2}$, $\sin\theta = \dfrac{1}{\sqrt{5}}$ (from Pythagoras)

$\therefore T = \dfrac{\sqrt{5}\,mg}{2}$ N



8 First consider the metal mould. Taking moments about point O,

$\frac{2}{3}\pi \times 60^3 \times \frac{3}{8} \times 60 - \frac{2}{3}\pi \times 40^3 \times \frac{3}{8} \times 40$

$= \left(\frac{2}{3}\pi \times 60^3 - \frac{2}{3}\pi \times 40^3\right)\bar{x} \Rightarrow$

$\bar{x} = \frac{975}{38} \approx 25.7$ (3 s.f.) along the symmetry axis. Taking moments about O when the mould is filled with plastic

$10\rho\left(\frac{2}{3}\pi \times 60^3 - \frac{2}{3}\pi \times 40^3\right)\bar{x} + \rho\left(\frac{2}{3}\pi \times 40^3\right) \times \frac{3}{8} \times 40$

$= \left(10\rho\left(\frac{2}{3}\pi \times 60^3 - \frac{2}{3}\pi \times 40^3\right) + \rho\left(\frac{2}{3}\pi \times 40^3\right)\right)\bar{X}$.

From which we find $\bar{X} = 25.2$ cm along the symmetry axis. Now we can find the angle that the plane face makes with the vertical

$\tan\theta = \dfrac{\bar{X}}{60} = 0.42 \Rightarrow \theta \approx 22.8^\circ$.